Then $\left(\frac{\partial f(\vec{k})}{\partial t}\right)_{\text{collisions}}$ is evaluated by taking the difference between the number of collisions into a volume element at $\vec{k}$ and the number out of it; by considering only transitions between states of equal energy, we obtain

$$\left(\frac{\partial f(\vec{k})}{\partial t}\right)_{\text{coll.}} = [\,1 - f(\vec{k})] \int f(\vec{k}')\, P_{k'k}\, dS' - f(\vec{k}) \int [\,1 - f(\vec{k}')]\, P_{kk'} dS' \quad . \tag{IV-17}$$

Here $P_{kk'}\, dS'$ is the probability per unit time of an electron making a transition from a state k to a state k', both of which lie on the same spherical surface of constant energy; dS' is an element of area about the state k'. The form of $P_{kk'}$ is given in Eq. (IV-23); $U_{kk'}$ and $U_{k'k}$, the transition probabilities between volume elements in k space, are equal by detailed balancing. Since we have assumed spherical energy surfaces, $|\nabla_k E|$ in Eq. (IV-23) is constant on a surface of constant energy and

$$P_{kk'} = P_{k'k} \quad . \tag{IV-18}$$

Then

$$\left(\frac{\partial f(\vec{k})}{\partial t}\right)_{\text{coll.}} = \int_{\text{Fermi sphere}} [f(\vec{k}') - f(\vec{k})]\, P_{kk'}\, dS' \quad . \tag{IV-19}$$

Substituting for $f(\vec{k}')$, $f(\vec{k})$ from Eq. (IV-13) and dividing through by $\tau(\vec{k}) \left(\frac{\partial f(\vec{k})}{\partial t}\right)_{\text{fields}}$, and using the expression for $\left(\frac{\partial f(\vec{k})}{\partial t}\right)_{\text{fields}}$ from Eq. (IV-16) we obtain

$$\frac{1}{\tau(\vec{k})} = \int_{\text{Fermi sphere}} \left[1 - \frac{\tau(\vec{k}')\, k'_z}{\tau(\vec{k})\, k_z}\right] P_{kk'}\, dS' \quad . \tag{IV-20}$$

Equation (IV-20) for $\tau(\vec{k})$ is an integral equation. In order to estimate the anisotropy of $\tau(\vec{k})$ we shall set $\tau(\vec{k}')/\tau(\vec{k}) = 1$ inside the integral. This may be regarded as the first step of an iteration procedure for finding τ. Since we chose our electric field (z axis) along the direction $\vec{k}$ for which we are computing $\tau(\vec{k})$ we have

$$k_z = k_s \quad \text{and} \quad k'_z = k_s \cos\theta \tag{IV-21}$$

where θ is the angle between k and k'. Let ϕ be the angle between the plane of k and k' and the z-x plane. Then

$$\frac{1}{\tau(\vec{k})} = k_s^2 \int_0^{\pi} d\phi \int_0^{2\pi} [1 - \cos\theta] \, |\sin\theta| \, P_{\vec{k},\phi}(\theta) \, d\theta \, . \tag{IV-22}$$

We have chosen these limits on θ and ϕ because of the possibility that $P_{\vec{k},\phi}(\theta) \neq P_{\vec{k},\phi}(-\theta)$; this possibility arises because for an arbitrary direction of $\vec{k}$ the section of the Brillouin zone for phonons centered on the tip of the k vector is not symmetrical about the line $\theta = 0$. A phonon may be available for a normal process with angle θ but not for one with the angle $-\theta$. We would like to evaluate the inner integral numerically and prefer to include the effect of asymmetries in it.

We now consider the form of $P_{\vec{k},\phi}(\theta)$. Perturbation theory gives

$$P_{\vec{k},\phi}(\theta) = \frac{1}{4\pi^2\hbar} \frac{|U_{kk'}|^2}{|\nabla_k E|_{k'}} \tag{IV-23}$$

where $U_{kk'}$ is the matrix element of the perturbing potential U taken between the initial electron state ψ_k and the final state $\psi_{k'}$ [10]; unit crystal volume is assumed. We shall be interested only in the perturbation due to lattice vibrations, since at room temperature these dominate the scattering of electrons. We then write

$$\begin{aligned} U(\vec{r}) &= \sum_{\vec{l}} V[\vec{r} - \vec{l} - \vec{R}(\vec{l})] - V(\vec{r} - \vec{l}) \\ &= -\sum_{\vec{l}} \vec{R}(\vec{l}) \cdot \nabla V(\vec{r} - \vec{l}) \, ; \end{aligned} \tag{IV-24}$$

where $V(\vec{r} - \vec{l})$ is the potential at $\vec{r}$ associated with the ion at lattice point $\vec{l}$ and $\vec{R}(\vec{l})$ is the displacement at $\vec{l}$. $V(\vec{r} - \vec{l})$ includes both the potential due to the ion core and to the electrons that shield the core. The wave functions are